

Indicative Problems for Maths Camps

Who's Lying?

You meet Raniya, Samiha and Tochi.

1. One of them is a liar. Everything they say is **not true**.
2. One is a truth-teller. Everything they say is **true**.
3. One is a flip-flopper. When they say something, it **could be true** and it **could be not true**.

Raniya says, "I am not a truth-teller".

Samiha says, "I am not a flip-flopper".

Tochi says, "I am not a liar".

Which of Raniya, Samiha and Tochi is a truth-teller? Who is the flip-flopper, and who is the liar?

At the Doughnut Shop

Coco goes to buy some doughnuts. At the doughnut shop, doughnuts come in boxes of three different sizes:

1. Boxes of 90 doughnuts for £20
2. Boxes of 60 doughnuts for £15
3. Boxes of 40 doughnuts for £12

Coco spends **exactly** £116. Could Coco have bought 500 or more doughnuts?

Can Coco buy more than 500 doughnuts and spend **less** than £116?

Solutions

Who's Lying?

- If Raniya were a truth-teller, then she would tell the truth about who she is, and she would not say "I am not a truth-teller", which is a lie
- Similarly, if Raniya were a liar, then she would lie about who she is, and she would not say "I am not a truth-teller", which is true
- So Raniyah must be a flip-flopper
- That means that Samiha is not a flip-flopper, as there's only one flip-flopper among the three
- But Samiha said "I am not a flip-flopper", which is true. So Samiha can't be a liar
- We now know that Samiha isn't a flip-flopper and she isn't a liar, so she must be a truth-teller
- That means Tochi is the liar. Although Tochi said, "I am not a liar", he was lying about that too!

At the Doughnut Shop

For the first part,

- Boxes of 90 and boxes of 60 doughnuts all cost a multiple of £5
- If Coco didn't buy any boxes of 40 doughnuts, then the cost of all the larger boxes would have to be £116, which isn't a multiple of £5
- Similarly, if Coco bought one or two boxes of 40 doughnuts, then the cost of all the larger boxes would have to be £104 or £92, which are also not multiples of £5
- So Coco bought at least three boxes of 40 doughnuts, giving her $40 \times 3 = 120$ doughnuts
- This leaves £80 to be spent
- How many doughnuts can Coco buy with each remaining £1 she has?

- When Coco buys a box of 90 doughnuts, she gets $4\frac{1}{2}$ doughnuts for every £1 she spends
- When she buys a box of 60 doughnuts, she gets 4 doughnuts for every £1 she spends
- When she buys a box of 40 doughnuts, she gets $3\frac{1}{3}$ for every £1 she spends
- So no matter which boxes she picks, she can never get more than $4\frac{1}{2}$ doughnuts with each £1 she spends
- That means that with £80, she can't ever get more than $£80 \times 4\frac{1}{2} = 360$ doughnuts
- Together with the 120 from before, this means she can buy at most 480 doughnuts, which is less than 500. So the answer is **no**

For the second part,

- Coco can buy five boxes of 90 doughnuts and one box of 60 doughnuts
- She spends $£20 \times 5 + £15 \times 1 = £115$, which is less than £116
- She gets $90 \times 5 + 60 \times 1 = 510$ doughnuts, which is more than 500. So the answer is **yes**